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Dynamic hedge ratio for stock index futures: application of threshold VECM

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This study represents one of the first papers in stock-index-futures arbitrage literature to investigate the effects of arbitrage threshold on stock index futures hedging effectiveness by using threshold vector error correction model (hereafter threshold VECM). Moreover, in contrast to prior studies focusing on examining case studies involving mature stock markets, this study not only adopts US S&P 500 stock market as the sample but also adds an analysis of one emerging stock market, Hungarian BSI and examines the differences between them. Finally, this investigation employs a rolling estimation process to examine the impact of arbitrage threshold behaviours on the setting of futures hedging ratio. The empirical findings of this study are consistent with the following notions. First, arbitrage behaviour reduces co-movement between futures and spot markets and increases the volatility of both futures and spot markets. Second, this article denotes the outer regime of futures-spot market for the case of Hungarian BSI (US S&P 500) as a crisis (an unusual) condition. Moreover, arbitrage threshold behaviours make remarkable (unremarkable) shift on optimal hedge ratio between two different market regimes for the case of Hungarian BSI (US S&P 500). Finally, the framework involving regime-varying hedge ratio designed in this study provides a more efficient futures hedge ratio design for Hungarian BSI stock market, but not for US S&P 500 stock market.

I. Introduction

This study considers spot-futures arbitrage threshold in establishing stock index futures hedge ratio. The concept of arbitrage threshold is presented as the following notions. Theoretically, spot and futures prices converge at maturity and there is no possibility of arbitrage. However, numerous studies have found that index-futures arbitrageurs only enter the market if the deviation from the equilibrium relationship becomes sufficiently large to compensate for transaction costs, and risk and price premiums. Restated, for arbitrageurs to profit, the difference between the futures and spot prices must be sufficient to pay the associated trading costs.

Building upon the above, Balke and Fomby (1997) represent one of the first studies to introduce the threshold cointegration model for capturing the nonlinear adjustment behaviour of spot-futures markets. The concept of combining nonlinearity and cointegration has generated considerable applied interest, including the following applications: Yadav et al. (1994), Martens et al. (1998), Lin et al. (2003), Root and Lien (2003), McMillan (2005) dealing with...

Unlike the above studies, this study adopts a new approach to questions regarding the link between the idea of arbitrage threshold and the establishment of dynamic stock index futures hedge ratio. Briefly, key questions include: whether the spot and futures prices are more or less correlated when arbitrage trading is triggered; whether the arbitrage operations increase the volatility/stability of the spot and futures markets; and whether the consideration of the point of arbitrage threshold helps investors design a more efficient hedge ratio?

Furthermore, this study employs rolling estimation to include recent market information and continuously repeat the work of model estimation and conducting a robust test of out-of-sample hedging performance for various alternatives. Our idea is meaningful. The in-sample performances of the alternatives hedging strategies give an indication of their historical performance. However, market participants are more concerned with how well they can do in the futures using alternative hedging strategies. Therefore, this study conducts out-sample hedging effectiveness tests via a rolling-estimation process.

Last but not least, compared to previous related studies, this study not only adopts US S&P500 stock index as one representative sample of mature markets but also includes an analysis of Hungarian BSI stock index as one representative sample of emerging markets and examines the differences between the two markets. The comparative analysis is meaningful. Briefly, a key feature of the nonlinear threshold model is to capture extreme, rare, large deviations from the futures-spot equilibrium relationship, which cause pronounced effects on the hedging ratio design. In contrast, traditional linear approaches provide a poor description of extreme events. Furthermore, in general, emerging stock markets experience more extreme crisis events compared to mature stock markets. A derivative question is, would the nonlinear threshold system show better/poorer performance on picturing emerging/mature stock markets? To our knowledge, few studies have studied the aforementioned nontrivial issues relating to research on risk hedging design.

Section II describes the establishment of optimal hedge ratio using no-threshold systems. To provide a contrast, Section III presents how to establish optimal hedge ratio via threshold systems. Section IV presents the empirical results and provides economic and financial explanations for them. Finally, Section V presents conclusions.

II. Establishing Optimal Hedging Ratio via a No-threshold System

Minimum-variance hedge ratio

The establishment of a hedge ratio with index futures is heavily dependent on three key variables: correlation between futures and spot returns and standard error of the spot and futures returns. Specifically, the hedge ratio that minimizes the variance of spot position can be expressed as,

\[
HR = \frac{\text{Cov}(\Delta S_t, \Delta F_t)}{\text{Var}(\Delta F_t)} = \rho_{SF} \times \frac{\sigma_{SS}}{\sigma_{FF}} \quad (1)
\]

where \( \Delta \) denotes the difference operator (such as, \( \Delta F_t = F_t - F_{t-1} \)), while \( F_t \) and \( S_t \) are the log prices of index futures and index spot at time \( t \), respectively. Additionally, \( \text{Cov}(\Delta S_t, \Delta F_t) \) and \( \text{Var}(\Delta F_t) \) are the covariance of \( \Delta S_t \) and \( \Delta F_t \) and the variance of \( \Delta F_t \), respectively. Moreover, \( \sigma_{SS} \) and \( \sigma_{FF} \) are the SDs of \( \Delta S_t \) and \( \Delta F_t \), respectively. Finally, \( \rho_{SF} \) is the correlation coefficient between \( \Delta S_t \) and \( \Delta F_t \).

This study examines whether investors can establish a more efficient hedge ratio after taking account of the idea of arbitrage threshold. Furthermore, what are the effects of arbitrage threshold on the three key parameters of the minimum-variance hedge ratio? To answer the above questions, this study adopts two common models without the threshold mechanism, including: (1) OLS (ordinary least squares) and (2) VECM (vector error correction model) as the two benchmarks. The following discussions outline the model specifications and potential limitations.

**OLS**

The variance-minimizing hedge ratio can be determined via simple OLS analysis,

\[
\Delta S_t = \alpha + \beta \cdot \Delta F_t + \epsilon_t; \quad \epsilon_t \sim \text{iid}(0, \sigma^2) \quad (2)
\]
The slope coefficient, $\beta$ in the above regression, is the same as the minimum-variance hedge ratio (namely, HR),

$$HR = \beta$$

(3)

However, the OLS model suffers two limitations. First, the OLS work assumes stability of the variances of futures and spot returns and the correlations between them. Second, the OLS system fails to account for the concept of cointegration, which captures the notion that nonstationary variables may nonetheless process long-run equilibrium relationships. In brief, the index futures and the underlying index tend to be equal in the long-term. The presence of transaction costs and other market imperfections may prevent economic agents from making continuous adjustments. Only when the deviation from the futures-spot equilibrium exceeds a critical threshold do the benefits of adjustment exceed the costs, and thus economic agents act to move the futures-spot system back towards the equilibrium.

### III. Establishing Optimal Hedging Ratio via a Threshold System

The presence of transaction costs and other market imperfections may create a band within which futures and spot prices are free to diverge. To capture the discrete adjustment process in the futures-spot markets, this study adopts the model of Balke and Fomby (1997), which used a two-step framework to examine the concept of arbitrage threshold in the futures-spot markets. In brief, the first step is to
examine whether threshold cointegration exists and determine the values of threshold parameters. The second step is then to run threshold VECM. The procedures involved in this two-step framework are briefly summarized below.

**Threshold cointegration**

As with the VECM, this study first regresses $F_t$ on $S_t$,

$$F_t = \lambda_0 + \lambda_1 \cdot S_t + Z_t$$

(8)

The next step is to address and examine the behaviour of threshold cointegration. $k$ is defined as an observable discrete regime variable, the threshold cointegration with thresholds is then established as follows,

$$Z_t = \eta_0^{k=1} + \eta_1^{k=1} \cdot Z_{t-1} + \epsilon_i^{k=1}, \quad \text{if} \quad |Z_{t-1}| \leq \theta$$

$$= \eta_0^{k=2} + \eta_1^{k=2} \cdot Z_{t-1} + \epsilon_i^{k=2}, \quad \text{if} \quad |Z_{t-1}| > \theta$$

(9)

where $\theta$ denotes the threshold parameter.

Apparently, two market regimes are defined in the above setting: (1) regime 1 or the central regime (namely $k=1$), is set where $|Z_{t-1}| \leq \theta$, while (2) regime 2 or outer regime (namely $k=2$), is set where $|Z_{t-1}| > \theta$. The dynamics of the mispricing term, namely $Z_t$, depend on the market regime where that term is located. In brief, in the central regime, $-\theta < Z_{t-1} < \theta$, the mispricing term, $Z_{t-1}$ is too small to cover the transaction costs of arbitrage trading, and thus arbitrage trading was not triggered. Consequently, no tendency exists for the index futures and spot prices to adjust back towards the equilibrium. The futures and spot prices are free to diverge, that is $Z_t$ is a nonstationary I(1) variable.

In contrast, in the outer regime $Z_{t-1} < -\theta$ or $Z_{t-1} > \theta$, the mispricing term, $Z_{t-1}$, is sufficiently large to support the transaction cost, and consequently arbitrage trading is triggered. The futures and spot prices are cointegrated and the $Z_t$ becomes a stationary autoregression.

Generally, one of the difficulties in operating threshold models is estimating the threshold parameter, namely $\theta$ in Equation 9. This study follows Balke and Fomby (1997) in designing grid-search procedures for estimating the threshold parameter. The procedures are presented as follows: (1) $F_t$ is regressed on $S_t$ and then the observations of equilibrium error, $Z_t$ are obtained; (2) a series of arranged error term is established that orders the observations of $Z_t$ according to the value of $Z_{t-1}$, rather than according to time; (3) by assigning two small numbers to serve as the initial value of $\theta$ and $-\theta$, for example 0.005 and $-0.005$, the series of arranged error terms can be split into two different regime areas, inside/outside the thresholds; (4) the regression of $Z_t$ on $Z_{t-1}$ is estimated for each regime area and the residual sum of squared (RSS) is calculated and saved; (5) sum up the value of RSS for each regime and save it; (6) the values of $\theta$ and $-\theta$ are increased using one grid with very small values of 0.0001 and $-0.0001$, and the above 4th and 5th procedure is then repeated for the new values of $\theta$ and $-\theta$; (7) procedures 4, 5 and 6 are then repeated and the value of sum of RSS for each regime is derived for each value of $\theta$ and choose the value of $\theta$ for which the sum of RSS for each regime is minimum.

**Threshold VECM**

Following obtaining the estimates of the threshold parameters from the threshold autoregression, namely $\theta$ in Equation 9, then the threshold VECM can be established as follows,

$$\Delta F_t = \alpha_F^{K=1} + \beta_F^{K=1} \cdot Z_{t-1} + \sum_{i=1}^{p} \gamma_{FF,i}^{K=1} \cdot \Delta F_{t-i} + \sum_{j=1}^{q} \gamma_{FS,j}^{K=1} \cdot \Delta S_{t-j} + \epsilon_{F,i}^{K=1}, \quad \text{if} \quad |Z_{t-1}| \leq \theta$$

(10)

$$= \alpha_F^{K=2} + \beta_F^{K=2} \cdot Z_{t-1} + \sum_{i=1}^{p} \gamma_{FF,i}^{K=2} \cdot \Delta F_{t-i} + \sum_{j=1}^{q} \gamma_{FS,j}^{K=2} \cdot \Delta S_{t-j} + \epsilon_{F,i}^{K=2}, \quad \text{if} \quad |Z_{t-1}| > \theta$$

$$\Delta S_t = \alpha_S^{K=1} + \beta_S^{K=1} \cdot Z_{t-1} + \sum_{i=1}^{p} \gamma_{SF,i}^{K=1} \cdot \Delta F_{t-i} + \sum_{j=1}^{q} \gamma_{SS,j}^{K=1} \cdot \Delta S_{t-j} + \epsilon_{S,i}^{K=1}, \quad \text{if} \quad |Z_{t-1}| \leq \theta$$

$$= \alpha_S^{K=2} + \beta_S^{K=2} \cdot Z_{t-1} + \sum_{i=1}^{p} \gamma_{SF,i}^{K=2} \cdot \Delta F_{t-i} + \sum_{j=1}^{q} \gamma_{SS,j}^{K=2} \cdot \Delta S_{t-j} + \epsilon_{S,i}^{K=2}, \quad \text{if} \quad |Z_{t-1}| > \theta$$

The idea of the above specification is presented as follows, in the central regime: $-\theta < Z_{t-1} < \theta$ (namely, $k=1$), the mispricing term, $Z_{t-1}$ is too small to trigger arbitrage trading; however, in the outer regime: $Z_{t-1} < -\theta$ or $Z_{t-1} > \theta$
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(namely, \( k = 2 \)), the mispricing term, \( Z_{t-1} \), is sufficiently large to initiate arbitrage trading and thus deviation from the equilibrium condition, namely \( Z_{t-1} \), helping to significantly effect next period prices of index futures and index spot. In brief, the parameters \( \beta_F \) and \( \beta_S \) are significant (insignificant) for the futures and spot equations, respectively.

Next, by considering two pairs of error terms, namely \( u_{F,t} \) and \( u_{S,t} \), then the two sets of covariance matrix are established as follows,

\[
\begin{bmatrix}
\sigma_{F}^K \\
\sigma_{S}^K
\end{bmatrix}
\sim \text{iid} \left( 0, \begin{bmatrix}
\sigma_{FF}^K & 0 \\
0 & \sigma_{SS}^K
\end{bmatrix} \right) \cdot \begin{bmatrix}
1 & \rho_{SF}^K \\
\rho_{SF}^K & 1
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
\sigma_{F}^K \\
\sigma_{S}^K
\end{bmatrix}
\sim \text{iid} \left( 0, \begin{bmatrix}
\sigma_{FF}^K & 0 \\
0 & \sigma_{SS}^K
\end{bmatrix} \right) \cdot \begin{bmatrix}
1 & \rho_{SF}^K \\
\rho_{SF}^K & 1
\end{bmatrix}
\]

By considering the above two classes of variances and correlation for two different market regimes, this study can then create a dynamic hedge ratio framework based on two different measures of hedge ratio, as follows,

\[
HR^k = \rho_{SF}^k \times \frac{\sigma_{S}^k}{\sigma_{F}^k},
\]

for the central regime: \(-\theta \leq Z_{t-1} \leq \theta\)

\[
HR^k = \rho_{SF}^k \times \frac{\sigma_{S}^k}{\sigma_{F}^k},
\]

for the central regime: \(Z_{t-1} < -\theta \) or \( Z_{t-1} > \theta \) (12)

Comparing the threshold VECM and the two benchmark models, namely, VECM and OLS, there are two key advantages for the threshold VECM. First, the discrete adjustments in the threshold system release the unrealistic assumption that a tendency to move a long-run equilibrium of the futures-spot market exists in every time period. Second, the threshold VECM considers the concept of arbitrage threshold and uses it to identify the outer/central regime at each time point using the data, and then calculates the three key parameters of hedge ratio for each regime. The threshold VECM can thus overcome the limitations of constant variance and correlation.

The idea of this article is meaningful and intuitive. Owing to arbitrage threshold behaviour, the relationship between time series of spot and futures prices would differ by regimes, and the same for variance and correlation of the futures-spot market and the hedge ratio. To summarize, designs characterized by regime-varying variance and correlation via threshold systems can help investors create a dynamic hedge ratio which can be flexibly adjusted according to various market states and thus help to eliminate unrealistic assumptions of constant variances and correlation.

IV. Empirical Results

The data in this study comprises of the daily stock index futures and spot prices for the US S&P 500 and the Hungarian BSI markets. This article uses the US S&P 500 stock (Hungarian BSI) market to represent a representative case of mature (emerging) markets. The period of data is from 3 January 1996 to 30 December 2005 for 2610 observations. All data are obtained from the Datastream database.

Preliminary comparative results for the two sets of markets

 Compared to previous studies, one feature of this study is to identify the differences between mature and emerging stock markets. The following discussions use data for the entire sample period and employ several regular tests and statistics for briefly investigating the differences/similarities between the two markets.

Table 1 lists the unit root and cointegration tests for the stock index futures and spot for both markets. The empirical results reveal that both series of futures and spot indices are nonstationary in both cases. However, the logarithmic first difference of stock index including futures and spot is stationary. Additionally, the cointegration test indicates that the error correction term, namely \( Z_n \), for the stock index futures and spot is stationary, a finding that is consistent for both cases. That is, the cointegration relationship of the price series for the futures and spot markets holds.1

1 This study uses the criterion of BIC for selecting the lag lengths for the ADF test (the author is thankful to the suggestion from an anonymous referee). Specifically, the setting with minimum BIC values is used for the ADF test with intercept. Taking the error correction term, namely \( z_n \), as an example, the setting with lag length numbers of 4 (2) is for the case of the US S&P 500 (Hungary BSI). Last but not the least, the conclusion from unit root and cointegration tests is robust for the setting with various lag length numbers.
Table 1. Unit root tests and cointegration tests of stock index futures and spot

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<tr>
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<tbody>
<tr>
<td>Log levels</td>
<td>−2.071</td>
<td>−2.075</td>
<td>−2.975</td>
<td>−3.051</td>
</tr>
<tr>
<td>Percentage of returns</td>
<td>−13.762*</td>
<td>−13.594*</td>
<td>−12.080*</td>
<td>−11.629*</td>
</tr>
<tr>
<td>Error correction term</td>
<td>−7.4845*</td>
<td>−10.43306*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: (1) The sample period is from 3 January 1996 to 30 December 2005, for 2610 daily observations. (2) The unit test for the log levels and return rates of futures and spot prices is Dickey and Fuller’s (1979) augmented Dickey–Fuller tests for unit roots (ADF). Cointegration tests are based on Engle–Granger (1987) procedure. The lag length selection in the ADF test is based on the criterion of the value of BIC. (3) * Denotes significance at 1%. Our empirical results show that both series of futures and spot indices are nonstationary for all cases. However, the change rates of stock index futures and spot are stationary. Besides, the cointegration test shows that the error correction term of stock index futures and spot is stationary and the finding is consistent for all cases. In other words, the price series of futures and spot are cointegrated.

Table 2 summarizes several basic statistics on the logarithmic first difference of the stock index futures and spot prices for both sample markets. Initially, the return mean is close to 0, the skewness coefficient is not equal to zero and the kurtosis coefficient is greater than 3 for all cases. In detail, for each date \( t \), 1500 historical data are adopted, namely \( \{A_{Si}, A_{Fi}, Z_{t_i} \}_{i=1}^{1500} \) for estimating the model parameters. The parameter estimates are then used to establish the out-of-sample hedge ratio for date \( t \). Adopting the 1st round estimation as an example, this study uses the data from \( t = 1 \) to \( t = 1500 \), to estimate our models and obtain the parameter estimates. Next, the error correction term, \( Z_{t_{1500}} \), and the estimate of threshold parameter, namely \( \theta \) are used to identify the market state for the next day. Specifically, \( -\theta \leq Z_{t_{1500}} \leq \theta \) \((Z_{t_{1500}} < -\theta \) or \( Z_{t_{1500}} > \theta \)), then the central (outer) regime is identified and thus the corresponding hedging ratio \( HR^{k=1} \) \((HR^{k=2}) \) is adopted for day \( t = 1501 \).

3 The value of kurtosis coefficient is one measure of the fatness of the tails of distribution.
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For convenience, this study sets the lag number order in the VECM and threshold VECM to unity, namely $p = 1$ and $q = 1$. The sample period in this article contains 2610 trading days. For the present tests involving a 1500 prior-trading-days estimation window and one-day as the order of the lagged term, 1109 out-of-sample estimates of hedge ratio are obtained.

Notably, to avoid the problem of a small number of observations for any particular regime, particularly the outer regime, it is necessary to restrict the value of the threshold parameters. This article uses the values of 2 and 20% percentiles of the error correction term, namely $Z_{t-1}$ as the boundary values of the threshold parameters. That is, the observation percentage for the outer regime is allowed to range from 2 to 20%. That is, for each repeated estimation work with 1500 daily data, there are 30 (300) observations for the outer regime at least (at most).

Arbitrage threshold and dynamic hedge ratio design

To concentrate on arbitrage threshold behaviour and examining its impact on hedge ratio establishment, the repeated rolling-estimation process focuses on the key parameters of the hedge ratio, and uses several figures to present our results.

Figure 1 shows the threshold parameter estimates and observation percentages of the outer regime for threshold VECM in the rolling estimation process. Notably, when running the rolling estimation process, few of our estimation results reach the boundary, namely 2 or 20% observations for the outer regime. Figures 2 and 3 show the estimates of three key parameters for the hedge ratio, (1) $\rho_{S,F}^k$, (2) $\sigma_{SS}$ and (3) $\sigma_{FF}$, in addition to the relative size of the SE of spot returns to futures returns, namely (4) $\sigma_{SS}/\sigma_{FF}$ for the threshold VECM. Apparently, the values of the estimates appear to be significantly different under various market regimes. Moreover, the estimates for the outer regime are substantially more volatile.

Table 3 presents the mean value and SE of four key parameter estimates for hedge ratio in the rolling process under various market regimes. First, in the US S&P 500 stock market, the average value of $\rho_{S,F}^{k=2}$ is 0.9678, which is smaller than the average value of $\rho_{S,F}^{k=1}$ for 0.9784. Moreover, the average value of $\rho_{S,F}$ for the VECM, namely the setting without threshold, is 0.9765, which is within range [0.9678–0.9784]. The conclusion is clear. Settings which ignore arbitrage threshold underestimate (overestimate) the magnitude of correlation between the futures and spot markets in the central (outer) market regime.

This phenomenon is explained below. According to the present empirical findings, the outer market regime is associated with a notable arbitrage behaviour, namely simultaneous short selling of the spot (future) index and purchase of the future (spot) index when the mispricing term, namely $Z_{t-1}$, is negative (positive). The arbitrage behaviour clearly causes spot and futures prices to tend to move in opposite directions and thus reduces the scale of co-movement between them.

Second, examine the magnitudes of SD of futures and spot markets during various market regimes. In the US S&P 500 stock market, the findings of this study indicate that the average values of $\sigma_{FF}^{k=2} (=0.0140)$ and $\sigma_{SS}^{k=2} (=0.0133)$ are greater than the estimates of $\sigma_{FF}^{k=1} (=0.0129)$ and $\sigma_{SS}^{k=1} (=0.0124)$, respectively. Furthermore, the values of $\sigma_{FF}^{k=1} (=0.0130)$ and $\sigma_{SS}^{k=1} (=0.0125)$ for the VECM lie within the range of $[\sigma_{FF}^{k=2}, \sigma_{FF}^{k=1}]$ and $[\sigma_{SS}^{k=2}, \sigma_{SS}^{k=1}]$, respectively. The conclusion is clear. The settings that do not incorporate the concept of arbitrage threshold will underestimate (overestimate) the magnitude of the volatility of the futures and spot markets during the outer (central) market regime. This finding is consistent with the notion that arbitrage behaviour between the futures and spot markets increases volatility in both markets.

Third, the estimates of $(\sigma_{SS}^{k=2}/\sigma_{FF}^{k=2})$ are smaller than $(\sigma_{SS}^{k=1}/\sigma_{FF}^{k=1})$, while the estimates of $(\sigma_{SS}/\sigma_{FF})$ lie between the above two values. Taking the US S&P 500 stock market as an example, the average values of $(\sigma_{SS}^{k=2}/\sigma_{FF}^{k=2})$, $(\sigma_{SS}^{k=1}/\sigma_{FF}^{k=1})$ and $(\sigma_{SS}/\sigma_{FF})$ are 0.9461, 0.9637 and 0.9620, respectively. This result indicates that the setting without threshold setting overestimates (underestimates) the relative size of the SE of the spot position to the futures position for the outer (central) regime. Furthermore, the aforementioned empirical findings are consistent

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3 Regarding the determination of lag number, the two most widely used criteria are AIC and BIC. However, in the rolling-estimation used here, each estimation work corresponds to different data periods. To our knowledge, the results of AIC and BIC are time-sensitive, and thus it is impossible to obtain a consistent conclusion regarding the lag number. Furthermore, the parameter numbers increase with the lag number. In detail, consider the threshold VECM with two states, there are additional $12(2 \times 2 \times 3)$ parameters in the setting with $p = 2$ and $q = 2$ relative to the setting with $p = 1$ and $q = 1$. Therefore, for convenience and to avoid over-parameter problems, this study adopts the simple case with a lag of one, namely $p = 1$ and $q = 1$.  

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Fig. 1. Threshold parameter estimates and observation percentage of outer regime in the rolling estimation process: US S&P500 and Hungarian BSI
Fig. 2. Key parameter estimates of hedge ratio in the rolling estimation process: US S&P500
Fig. 3. Key parameter estimates of hedge ratio in the rolling estimation process: Hungarian BSI
with the case of the Hungarian BSI market. The finding is consistent with the notion that arbitrage trading increases volatility in both futures and spot markets; however, the effects are greater in the futures markets. One explanation for this phenomenon is that the futures market can be considered as a superior vehicle compared to the spot markets because of lower trading costs, fewer limitations on short sales and higher leverage effect because of superior vehicle compared to the spot markets.

Notably, in the rolling estimation process, few inconsistent results occur. Take Fig. 2(b), the SE of futures position, namely \( \sigma_{FF}^k \) in the US S&P 500 market, as an example. In the rolling estimation process, the estimates of \( \sigma_{FF}^{k=2} \) for the outer regime are significantly greater than the \( \sigma_{FF}^{k=1} \) for the central regime for most situations. Although the estimates of \( \sigma_{FF}^{k=2} \) are smaller than the \( \sigma_{FF}^{k=1} \) for some cases, however, the proportion is relatively low. Specifically, for the 1109 out-sample results, the proportion of \( \sigma_{FF}^{k=1} \) exceeding \( \sigma_{FF}^{k=2} \) is 258-in-1109 or 23.2%. Apparently, the smidge is less than average.

Last but not the least, the SE of the rolling estimates for the outer regime is significantly greater than the one for the central regime. This finding states that all the parameter estimates are more unstable (stable) in the outer (central) regime.\(^4\)

**Comparison of the hedging effectiveness of various alternatives**

The present results show that all the key parameters for hedge ratio perform different pattern under various market regimes. The derivative question is, could the regime-varying hedge ratio design derived from the threshold VECM help investors to establish a more efficient hedge strategy?

For each date, this study uses the pre-1500 daily data to estimate the model parameters and three key parameters of the minimum-variance hedge ratio. Next, this study establishes the minimum-variance hedge ratio for the next day observation. Figure 4 presents the out-sample hedge ratio estimates using the threshold VECM for the two cases: US S&P 500 and Hungarian BSI markets. Remarkably, the hedge ratio estimate in the outer regime, namely \( HR^{k=2} \), is smaller than that in the central regime, namely \( HR^{k=1} \). Moreover, the hedge ratio estimate via the VECM, namely the no-threshold system, lies within the range of \( HR^{k=1} \) and \( HR^{k=2} \). This finding is consistent with the notion that the system that does not consider the point of arbitrage threshold will overestimate (underestimate) the optimal hedge ratio during the outer (central) market regime.\(^5\)

Regarding the finding that \( HR^{k=1} \) exceeds \( HR^{k=2} \), this study provides the following explanations.

\(^4\) See Li and Lin (2004) for the related discussions.

\(^5\) Undeniably, a smidge of inconsistent results occurs in the rolling estimation process. However, it is much less than average. In detail, for the 1109 out-sample results, there is the proportion of 80-in-1109 or 7.2% (105-in-1109 or 9.5%) for the situation of \( HR^{k=2} \) exceeding \( HR^{k=1} \) in the case of US S&P 500 (Hungarian BSI).
The empirical results of this study show that the correlation coefficient between the futures and spot markets in the central regime (or $\rho_{S,F}^{k=1}$) is greater than that in the outer regime (or $\rho_{S,F}^{k=2}$). Additionally, the relative size of the SE of the spot position to the futures position in the central regime (or $\sigma_{SS}^{k=1}/\sigma_{FF}^{k=1}$) exceeds that in the outer regime (or $\sigma_{SS}^{k=2}/\sigma_{FF}^{k=2}$). Therefore, the $HR^{k=1} = \rho_{S,F}^{k=1}(\sigma_{SS}^{k=1}/\sigma_{FF}^{k=1})$ for the central regime will be greater than the $HR^{k=2} = \rho_{S,F}^{k=2}(\sigma_{SS}^{k=2}/\sigma_{FF}^{k=2})$ for the outer regime.

Next, this study examines whether investors can use the framework which considers the arbitrage threshold behaviours to establish a more efficient hedge ratio. Table 4 lists out-sample hedging effectiveness of alternative specifications for US S&P 500 and Hungarian BIS stock markets. In detail, the variance (namely, Var) of the hedged spot position with index futures can be presented as,

$$\text{Var}(\Delta S_i - HR \cdot \Delta F_i)$$

where $\Delta S_i$ and $\Delta F_i$ represents the rates of change of stock index futures and spot, respectively, and HR represents the minimum-variance hedged ratio. Moreover, for benchmark purpose, this article considers an unhedged spot position and uses it to
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Table 4. Hedging effectiveness of regime-switching hedge ratio via threshold VECM against alternative no-threshold models

<table>
<thead>
<tr>
<th></th>
<th>US S&amp;P 500</th>
<th>Hungarian BSI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Variance</td>
<td>Variance reduction improvement (%)</td>
</tr>
<tr>
<td>Unhedged</td>
<td>1.141207</td>
<td>–</td>
</tr>
<tr>
<td>OLS</td>
<td>0.043041</td>
<td>96.22848</td>
</tr>
<tr>
<td>VECM</td>
<td>0.042324**</td>
<td>96.29133*</td>
</tr>
<tr>
<td>Threshold VECM</td>
<td>0.042629</td>
<td>96.2646</td>
</tr>
</tbody>
</table>

Notes: (1) The hedge ratio that minimizes the variance of spot position can be expressed as:

$$HR = \frac{\text{Cov}(\Delta S_t, \Delta F_t)}{\text{Var}(\Delta F_t)} = \rho_{S,F} \times \frac{\text{SD}(\Delta S_t)}{\text{SD}(\Delta F_t)}$$

where $\Delta S_t$ and $\Delta F_t$ denote rates of change of prices of futures and spot returns, respectively. COV($\Delta S_t$, $\Delta F_t$) and Var($\Delta F_t$) are the covariance of $\Delta S_t$ and $\Delta F_t$ and the variance of $\Delta F_t$, respectively. Moreover, SD($\Delta S_t$) and SD($\Delta F_t$) are the SDs of $\Delta S_t$ and $\Delta F_t$, respectively. $\rho_{S,F}$ is the correlation coefficient between $\Delta S_t$ and $\Delta F_t$.

(2) The variance (namely, Var) of the hedged spot position with index futures can be presented as:

$$\text{Var}(\Delta S_t - HR \cdot \Delta F_t)$$

where $\Delta S_t$ and $\Delta F_t$ represents the rates of change of stock index futures and spot, respectively, and HR represents the minimum-variance hedge ratio.

(3) For benchmark purpose, this article considers an unhedged spot position and uses it to calculate the risk reduction improvement percentage for various alternatives: Risk reduction improvement% $= -100 \times \frac{\text{risk of hedged spot position via specific alternative} - \text{risk of unhedged spot position}}{\text{risk of unhedged spot position}}$.

(4) * and ** Represents the maximum (minimum) value in the column.

Notably, for the case of Hungarian BSI market, the empirical results of this study indicate that the risk of the hedged spot position established via the threshold VECM is smaller than that of other alternatives. Moreover, compared to unhedged position, the threshold VECM hedge strategy provides investors risk reduction improvement percentage by 80.78% which is greater than the values of 73.08 and 78.65% for OLS and VECM, respectively. However, for the case of US S&P 500 market, the threshold systems cannot help investors to design a better hedge ratio with less risk. The empirical findings presented in this study reveal that the risk of hedged spot position via the VECM, namely the setting without a threshold, is smaller than for the setting with a threshold.

Several explanations are provided for this phenomenon as follows. First, refer to Fig. 1 for the rolling estimates of threshold parameter, namely $\theta$ in the threshold VECM. Taking the average value as a criterion, the average value of $\theta$ estimates in the rolling estimation process is 0.0066 and 0.0322 for the case of US S&P 500 and Hungarian BSI, respectively. Apparently, the latter is significantly greater than the former and the latter is near 4.8 ($=0.0322/0.0066$) times of the former. This finding is consistent with the notion that emerging stock markets, like the case of Hungarian BSI, are much more immature, and thus the magnitude of deviation between the futures and spot prices is considerably much higher.

Consequently, this article defines the outer regime in the case of Hungarian BSI as a crisis condition in contrast with an unusual condition for the case of US S&P 500.6

Furthermore, refer to Fig. 4 for the rolling hedge ratio estimates under various market regimes. In the case of Hungarian BSI, the average value of the hedge ratio in the outer regime, namely $HR^{k=2}$ is 0.4775, in contrast with that in the central regime, namely $HR^{k=1} = 0.7825$. The percentage difference equals approximately 64% ($(0.7825 - 0.4775)/0.4775$). Nevertheless, for the case of US S&P 500, the $HR^{k=2}$ for the outer regime is 0.9158 while

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6 See Li (2005) for the related discussions.
the HR^{k=1} for the central regime is 0.9430. The percentage difference between the two measures is just 2.96% ((0.9430 – 0.9158)/0.9158) and is clearly smaller than the value for the case of Hungarian BSI. The more/less significant shift in the hedge ratio between two various market regimes is consistent with the finding of crisis/unusual condition in the outer regime for the case of Hungarian BSI/US S&P 500. Moreover, the above characteristics also provide explanations for remarkable/unremarkable risk hedging effectiveness performance for the threshold VECM in the case of Hungarian BSI/US S&P 500.

Last but not the least, we provide two robustness tests for our empirical results. The first test is for the selection of window length of data in the rolling estimation process. This study uses the 1500-day windows, considerably longer than the 250-day windows, the minimum requirement proposed by the Basle Committee, to capture the extreme and rare price movements in the outer regime.

Intuitively, the longer (shorter) window length, the more (less) capable to picture the extreme price movement processes at the outer regime, but the less (more) observations for the out-of-sample hedging performance tests. For examining the sensitivity of the hedging performances of threshold VECM to the window length choice, this study thus adopts five different settings for the estimation windows: 2000, 1750, 1500, 1250 and 1000 days. Furthermore, for case of the US S&P 500, the measures of variance reduction improvement percentage of the threshold VECM are 94.48, 96.15, 96.26, 95.05, 94.68 and 94.34% for the window length of 2000, 1750, 1500, 1250 and 1000 days, respectively. Although the threshold VECM with the windows length of 1500 days could provide investors the maximum value of variance reduction improvement percentage, however, the differences among the settings with various windows lengths are considerably small.

The second sensitivity test is for the selection of lag lengths in the threshold VECM. Undeniably, this study adopts an arbitrary small number of lags, namely p = 1 and q = 1, for the threshold VECM. By intuition, one might concern whether the hedging performances of the threshold VECM would be affected by the choice of lag numbers. This investigation thus adopts four possible settings in the lag numbers: p = q = 1, p = q = 2, p = q = 3 and p = q = 4. Nonetheless, the hedging performances of various alternatives are considerably insensitive to the choice of lag numbers. Specifically, in the case of the US S&P 500, the measures of variance reduction performance percentage are 96.2646, 96.2515, 96.2652 and 96.2717% for the threshold VECM with p = q = 1, p = q = 2, p = q = 3 and p = q = 4, respectively. Although the setting with higher lag numbers, namely p = q = 4, could provide a greater value of variance reduction performance percentage, however, the differences are trivial. Specifically, the difference of variance reduction performance percentage between the settings with one lag and four lags are 0.0071% only (=96.2717 – 96.2646%).

V. Conclusions

This investigation is one of the first studies on the systematic application of threshold systems to dynamic hedge ratio on stock index futures. Previous studies typically demonstrated the occurrence of arbitrage threshold in spot-futures markets. This study consistently demonstrated that arbitrage operations frequently occur in the outer regime. We posited that the incorporation of historical data with controlling regime changes owing to arbitrage threshold can contribute to the establishment of a futures hedge ratio.

The empirical findings of this study are consistent with the following notions. First, the arbitrage behaviours between the futures and spot markets will increase the volatility in these two markets. Second, the outer (central) regime will be associated with a smaller (greater) correlation coefficient between the futures and spot markets and a smaller (greater) value of the optimal hedge ratio. Third, this article defines the outer regime identified by threshold VECM as a crisis (unusual) state for the case of Hungarian BSI (US S&P 500). Finally, the present findings lend support to the superiority of the threshold VECM in enhancing hedging effectiveness for emerging markets such as the Hungarian BSI market, but not for developed markets such as the US S&P 500 market.

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References

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